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Reliability aspects of a series load–sharing system

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ABSTRACT: In reliability engineering, load–sharing is typically associated with a system in parallel configuration. Examples include bridge support structures, electric power supply systems, multiprocessor computing systems, etc. We consider a reliability maximization problem for a high–voltage commutation device, wherein the total voltage across the device is shared by the components in series configuration. Here, the increase of the number of load–sharing components increases component–level reliability (as the voltage load per component reduces) but may decrease system–level reliability (due to the increased number of components in series). We review optimal solutions for the proportional hazard and accelerated life models with the underlying exponential & Weibull distributions and elaborate on the log–linear, power, and Eyring laws used in the life–load models.

1 INTRODUCTION

A load–sharing system is typically associated with a system in parallel configuration. In their renowned text on System Reliability Theory, Rausand and Hoyland (2009) write "Consider a parallel system with two identical components. The components share a common load." Another famous text by Kapur and Lamberson (1977) also treats load–sharing systems exclusively as parallel systems.

Examples of parallel load–sharing systems include but are not limited to: civil engineering [e.g., structures (Chen & Lui 2005)], materials engineering [e.g., composites with fiber bundles (Phoenix & Tierney 1983)] power engineering [e.g., distributed generation systems (Marwalli & Keyhani 2004)], computer/network engineering [multiprocessor computing systems (Eager et al. 1986)]. Load–sharing system are also often discussed in the context of (still parallel) systems with k–out–of–n configuration, e.g., Huang & Xu (2010) and Amari & Bergman (2008).

In many applications of electrical and power engineering, namely in high voltage power equipment, one often runs into problem of switching high voltages by solid–state devices, where the system voltage (10–100 kV) well exceeds the operating voltages of individual switching components (1–3 kV). One of the commonly practiced ways to increase the voltage capability of high voltage switching devices is to put single switching components in a series configuration. Shown in Figure 1 is a high voltage thyristor switch (Gurevich & Krivtsov 1991), wherein the total switching voltage is shared by the serially connected thyristors.

In this case, the increase of the number of thyristors increases thyristor–level reliability (as the voltage load per thyristor reduces) but may decrease system–level reliability (due to the increased number of components in series). Clearly, the system reliability function in this case should have a maximum associated with an optimal number of thyristors in series.

We derived (Krivtsov et al. 2017) optimal solutions to the two popular life–load models: the

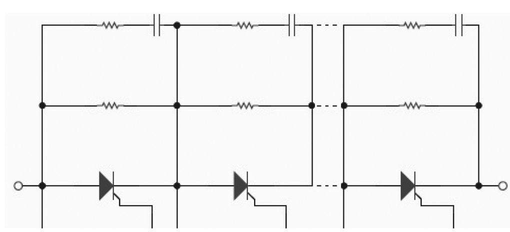

Figure 1. A serial load–sharing system, wherein total commutating voltage is shared by the serially connected SCRs (thyristors).

proportional hazard and the accelerated failure time with the underlying exponential and Weibull distributions. In this paper, we elaborate on the log–linear, power and Eyring laws used in the aforementioned life–load models.

2 PRELIMINARIES

Let $h(t;L)$ be the load–dependent failure rate of a component. Two commonly used models relating failure time to a load level are: the Proportional Hazard Model (PHM) and the Accelerated Failure Time Model (AFTM).

2.1 *Proportional hazard model*

Under the PHM, we have:

$$h(t;L) = \psi(L)\cdot h(t;L_0) = \psi(L)\cdot h_0(t), \tag{1}$$

where $h_0(t)$ is the baseline failure rate and $\psi(L_0)=1$.

The commonly used model for $\psi(L)$ is the log–linear (exponential) law: $\psi(L)=\exp(\alpha_0+\alpha_1 L)$. In fact, the exponential term can be replaced by any known positive, non–decreasing function. For example, for the log–linear law:

$$\psi(L)=\exp(\alpha_0+\alpha_1 L)=\delta\cdot\exp(\alpha_1 L) \tag{2}$$

For the power law:

$$\psi(L) = \delta\cdot \mathrm{L}^{\alpha} \tag{3}$$

For the linear law:

$$\psi(L) = c+\alpha L \tag{4}$$

For the Eyring law:

$$\psi(L)=L^{-1}\exp(\alpha_0+\alpha_1/L) \tag{5}$$

The baseline failure rate can follow any time–varying function. Further, for the cumulative hazard and the reliability functions, we have:

$$H(t;L) = \psi(L)\cdot H_0(t) \tag{6}$$

$$R(t;L)=\exp\{-H(t;L)\}=\left[R_0(t)\right]^{\psi(L)} \tag{7}$$

2.2 *Accelerated failure time model*

Under this model, the effect of the load is multiplicative in time. The reliability function is expressed as:

$$R(t;L) = R_0(t\cdot\phi(L)) \tag{8}$$

where $R_0(.)$ is the reliability function at the baseline load. Function $\phi(L)$ represents the acceleration factor at load L. Without loss of generality, we can assume that $\phi(L_0)=1$. When there is only one type of load, commonly used forms of $\phi(L)$ include the power law:

$$\phi(L) = \delta\cdot L^{\alpha} \tag{9}$$

the log–linear law:

$$\phi(L) = \delta\cdot\exp(\alpha L) \tag{10}$$

and the Eyring law:

$$\phi(L)=L^{-1}\exp(\alpha_0+\alpha_1/L) \tag{11}$$

Finally, for the hazard functions we have:

$$H(t;L) = H_0(t\cdot\phi(L)) \tag{12}$$

$$h(t;L) = \phi(L)\cdot h_0(t\cdot\phi(L)) \tag{13}$$

It must be noted that if the baseline distribution is Weibull (or Exponential) and the multiplicative factor (acceleration factor) follows the power law, then the AFTM and PHM coincide. However, in general, there is no direct duality between the models.

2.3 *System reliability under series load–sharing*

Consider a load–sharing series system with n components. The total load on the system is $L_T \equiv V$. The load is equally distributed between the components. Hence, the load on each component is:

$$L \equiv \frac{L_T}{n} = \frac{V}{n} \tag{14}$$

and the system's reliability is:

$$R_s(t;V,n)=\left[R(t;V/n)\right]^n \tag{15}$$

Note that for fixed t, function $R(t;V/n)$ is an *increasing* function in n. However, function R^n is a *decreasing* function in n. Hence, there is an optimal value of n that maximizes the system reliability.

Alternatively, one can consider a logarithmic function of system reliability:

$$\ln R_s(t;V,n) = \mathrm{n}\cdot\ln R(t;V/n) \tag{16}$$

The right hand side of the above equation has two product terms. The first term increases with n and the second term decreases with n. Further, it implies that

$$H_s(t;V,n) = n \cdot H(t;V/n) \tag{17}$$

To maximize the system reliability, we need to minimize the corresponding cumulative hazard function. Again, it has two product terms. The first term increases with the number of components and the second term decreases with the number. Hence, there exist an optimal value that minimizes the cumulative hazard function and maximizes the system reliability.

3 OPTIMAL NUMBER OF COMPONENTS IN A SERIES LOAD–SHARING SYSTEM

3.1 *Load–sharing under PHM*

The system reliability in this case is:

$$R_s(t;V,n) = \left[R(t;L) \right]^n \equiv \left[R\left(t;\frac{V}{n} \right) \right]^n = \left[\left[R_0(t) \right]^{\psi(V/n)} \right]^n = \left[R_0(t) \right]^{n\cdot\psi(V/n)} \tag{18}$$

It follows that for fixed t, $R_0(t)$ is also fixed. Hence, for fixed t, to maximize the system reliability, we need to minimize $g(n) \equiv n \cdot \psi(V/n)$. Thus, the optimal n is independent of the form of the underlying failure time distribution $R_0(t)$ and is also independent of mission time t.

Under the log–linear law, we have:

$$\psi(L) = \exp(\alpha_0 + \alpha_1 L) = \delta \cdot \exp(\alpha_1 L) \tag{19}$$

For notational simplicity, hereafter we'll use α in the place of α_1. Hence,

$$\psi(L) \equiv \psi(V/n) = \delta \cdot \exp(\alpha \cdot V/n) \tag{20}$$

$$g(n) \equiv n \cdot \psi\left(\frac{V}{n} \right) = n\delta \cdot \exp(\alpha \cdot V/n) \tag{21}$$

The minimum of $g(n)$ can be obtained as

$$\frac{d}{dn} g(n) \equiv \delta \cdot \exp\left(\alpha \cdot \frac{V}{n} \right) + n\delta.\exp\left(\alpha \cdot \frac{V}{n} \right)\left(-\frac{\alpha V}{n^2} \right)$$
$$\frac{d}{dn} g(n) = 0 \Rightarrow n = \alpha \cdot V \tag{22}$$

Thus, the optimal n that maximizes system reliability is

$$n = \left\| \alpha \cdot V \right\| \tag{23}$$

where ||.|| is the *nearest integer function.*

Figure 2 shows system–level reliability as a function of the number of the load–sharing components in series (with $\delta = 1$, $\alpha = 2$ and $V = 3$) for various values of component reliability. As expected, the optimal number does not depend on the latter.

It can be shown that for a PHM in the power law form, depending on model parameters, the optimal n will be either 1 or ∞. The linear law has a similar behavior. In both cases, the cost function can be used as regularization.

3.2 *Series load–sharing under AFTM*

The system reliability in this case is:

$$R_s(t;V,n) = \left[R(t;L) \right]^n \equiv \left[R_0(t \cdot \phi(L)) \right]^n = \left[\exp\left\{ -H_0(t \cdot \phi(L)) \right\} \right]^n \tag{24}$$

$$R_s(t;V,n) = \exp\left\{ -n \cdot H_0(t.\phi(L)) \right\} = \exp\left\{ -H_s(t;V,n) \right\} \tag{25}$$

where

$$H_s(t;V,n) = n \cdot H_0(t \cdot \phi(L)) \tag{26}$$

From the equation above, it follows that a) maximizing the system reliability is equivalent to minimizing the cumulative hazard function and b) unlike the PHM, the optimal value *does*

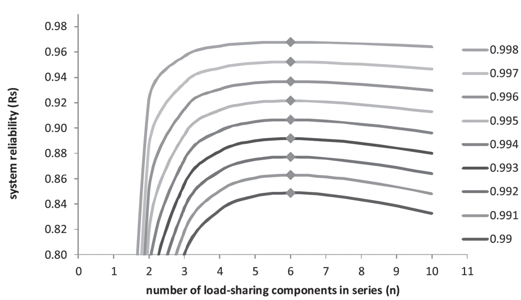

Figure 2. System–level reliability as a function of the number of the load–sharing components in series (with $\delta = 1$, $\alpha = 2$ and $V = 3$) for various values of component reliability. Optimal n is marked with the asterisk.

depends on the form of the underlying failure time distribution.

3.2.1 *AFTM with power law and the underlying exponential distribution*

For the exponential distribution, we have:

$$H_0(t) = \lambda t \tag{27}$$

Hence,

$$H_s(t;V,n) = n \cdot \lambda t \cdot \phi(L) = n \cdot \lambda t \cdot \phi(V/n) \tag{28}$$

In this case, maximizing the system reliability is equivalent to minimizing $g(n) \equiv n\phi(V/n)$. The optimal n is independent of the mission time or even the baseline hazard function.

Now, for the power law:

$$\phi(L) = \delta \cdot L^{\alpha} \tag{29}$$

and

$$g(n) = n\phi\left(\frac{V}{n}\right) = n\delta\left(\frac{V}{n}\right)^{\alpha} \tag{30}$$

Similar to the PHM with the power law, depending on model parameters, the optimal n will again be either 1 or ∞. The cost function can be used to regularize this case.

3.2.2 *AFTM with log–linear law and the underlying exponential distribution*

For the log–linear law:

$$\phi(L) = \delta \cdot \exp(\alpha L) \tag{31}$$

and

$$g(n) = n\phi\left(\frac{V}{n}\right) = n\delta \cdot \exp\left(\alpha \cdot \frac{V}{n}\right) \tag{32}$$

Note that the functional form of $g(n)$ is the same as in the PHM model with the log–linear law. Thus, the optimal n that maximizes system reliability is

$$n = \|\alpha \cdot V\| \tag{33}$$

3.2.3 *AFTM with Eyring law and the underlying exponential distribution*

$$\phi(L) = L^{-1}\exp(\alpha_0 + \alpha_1 / L) \tag{34}$$

and

$$g(n) = n\phi\left(\frac{V}{n}\right) = \frac{n^2}{V}\exp\left(\alpha_0 + \alpha_1\frac{n}{V}\right) \tag{35}$$

The minimum of $g(n)$ is $\arg\left(\frac{d}{dn}g(n) = 0\right)$. Let $X(n) \equiv \exp\left(\alpha_0 + \alpha_1\frac{n}{V}\right)$. Then with $\frac{d}{dn}g(n) \equiv \frac{X(n)}{V}(2n + \alpha_1)$, it becomes easy to show that the optimal n that maximizes system reliability is

$$n = \left\|-\frac{1}{2}\right\| \tag{36}$$

3.2.4 *AFTM with power law and the underlying Weibull distribution*

For the Weibull distribution, we have:

$$H_0(t) = \left(\frac{t}{\eta}\right)^{\beta} \tag{37}$$

Hence,

$$H_s(t;V,n) = n \cdot \left(\frac{t}{\eta}\right)^{\beta} \cdot \left[\phi(V/n)\right]^{\beta} \tag{38}$$

From the equation above, it follows that a) maximizing the system reliability is equivalent to minimizing $g(n) \equiv n \cdot [\phi(V/n)]^{\beta}$, and b) the optimal value of n is independent of the mission time and the scale parameter of the Weibull distribution.

For the power law:

$$\phi(L) = \delta \cdot L^{\alpha} \tag{39}$$

and

$$g(n) \equiv n\left[\phi\left(\frac{V}{n}\right)\right]^{\beta} = n\left[\delta \cdot \left(\frac{V}{n}\right)^{\alpha}\right]^{\beta} \tag{40}$$

Again, depending on model parameters, the optimal n will again be either 1 or ∞. The cost function can be used to regularize this case.

3.2.5 *AFTM with log–linear law and the underlying weibull distribution*

For the log–linear law:

$$\phi(L) = \delta \cdot \exp(\alpha L) \tag{41}$$

and

$$g(n) \equiv n\left[\phi\left(\frac{V}{n}\right)\right]^{\beta} = n \cdot \delta^{\beta} \cdot \exp\{\alpha\beta V/n\} \tag{42}$$

Hence,

$$\frac{d}{dn}g(n) = 0 \Rightarrow n = \alpha \cdot \beta \cdot V \tag{43}$$

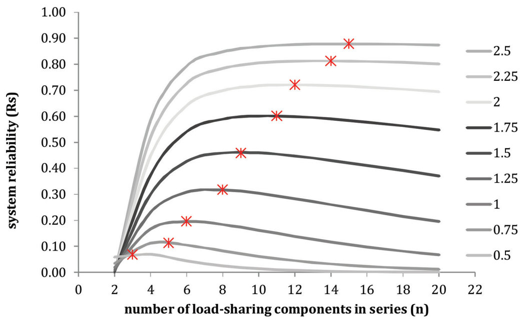

Figure 3. System–level reliability as a function of the number of the load–sharing components in series (with α = 2 and V = 3) for various values of Weibull shape parameter with the scale parameter of $\eta = 1$ and mission time $t = 0.1$. Optimal n is marked with the asterisk.

Thus, the optimal n that maximizes the system reliability is:

$$n = \|\alpha \cdot \beta \cdot V\| \tag{44}$$

Figure 3 shows system–level reliability as a function of the number of the load–sharing components in series (with $\alpha = 2$ and $V = 3$) for various values of Weibull shape parameter with the scale parameter of $\eta = 1$ and mission time $t = 0.1$.

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